National Aeronautics and Space Administration

Operational Implementation of a Pc Uncertainty Construct for Conjunction Assessment Risk Analysis

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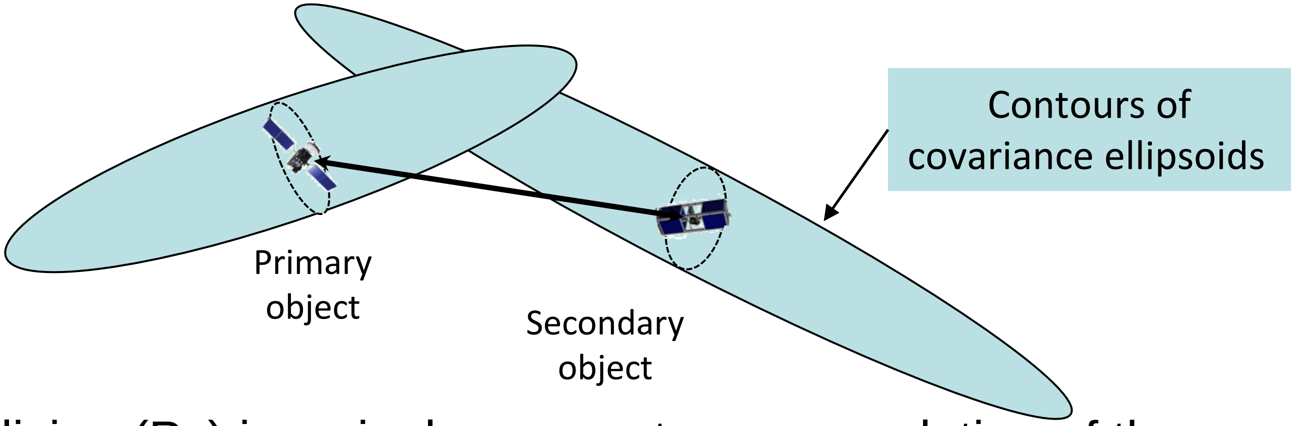
Agenda

- Introduction
- Error sources in probability of collision (Pc) computation
  - Uncertainty in primary and secondary object covariances
  - Uncertainty in hard-body radius (HBR) computation
- Covariance uncertainty computational methodology and testing results against operational dataset
- HBR uncertainty computational methodology and testing results against operational dataset
- Summary of effects on operational decisions

Background: Conjunction Assessment

- Conjunction Assessment Risk Analysis (CARA)
  - Evaluate collision risk between two conjuncting objects
  - Mitigate collision risk, if necessary



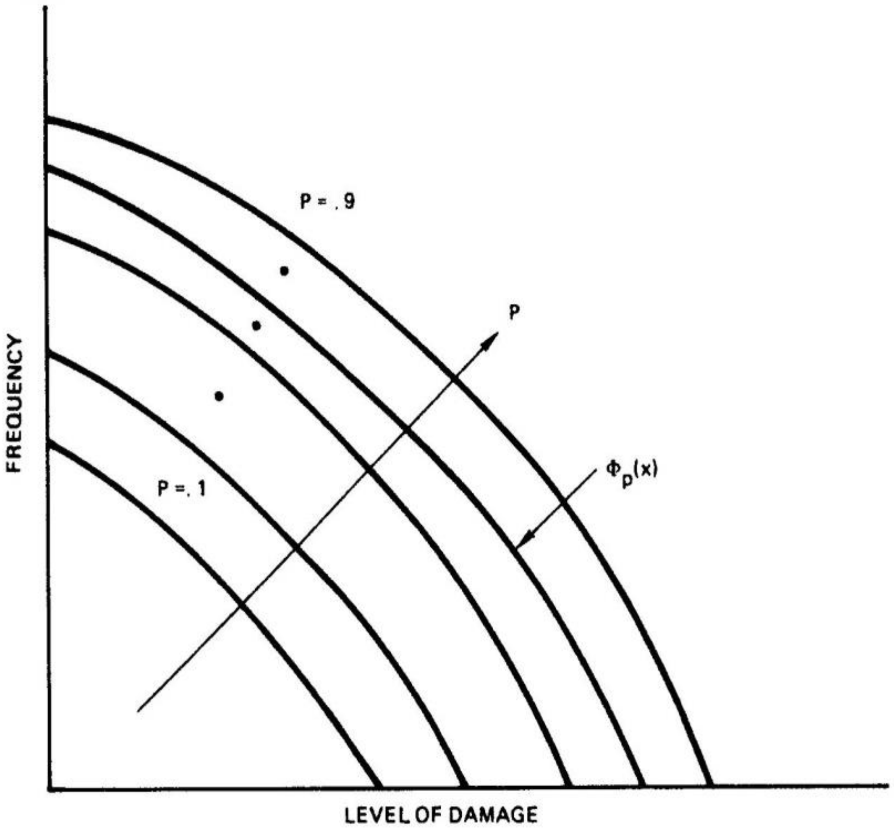
- Probability of Collision (Pc) is a single-parameter encapsulation of the risk and is computed from
  - Miss distance at time of closest approach (TCA)
  - State estimation error (covariance) for both objects
  - Hard-body radius (HBR) of both objects

Pc Error Sources

- Standard Pc calculation methods do not consider uncertainties in the inputs
  - Do not generate error statements about the result
  - Do not generate a probability density function (PDF) of Pc values
  - Nominal Pc represents risk but is just a point estimate
- Input uncertainties include
  - Uncertainties in covariances: non-captured orbit determination (OD) uncertainties at the OD epoch time and additional errors from covariance propagation
  - Uncertainties in HBR: variation in the projected areas of primary and secondary objects in the conjunction plane
  - Additional second-order error sources should also be considered
- All of these uncertainties should be characterized and included in the calculation so that the range of possible Pc values, not just a simple point estimate, can feed the risk analysis process

Probability Uncertainty and Risk Assessment

- Kaplan's second element of risk
  - Risk curve is relationship between frequency (likelihood) and consequence
  - Each curve is really an entirely family of curves (probability density) that reflect the uncertainties of the inputs in the calculation of likelihood and consequence
  - Diagram at right shows risk curve family as a series of probability contours
- To align with this construct, instead of single Pc value need PDF of values



Kaplan, S. and Garrick, B. "On the Quantitative Definition of Risk." *Risk Analysis*, Vol. 1 No. 1, pp. 11-27.

Methodology

- NASA CARA previously presented paper outlining a calculation technique for incorporating uncertainty sources into the Pc calculation*
  – Prototype tool developed and is in evaluation phase
- Incorporates the following uncertainty sources
  – Uncertainty in primary and secondary object covariances
  – Uncertainty in primary and secondary object HBR
  – Overall sampling uncertainty of Pc calculation
- Theory developed in previous paper; given only in summary here
- Present effort will focus on performance of first two uncertainty sources
  – Primary/secondary covariances
  – Primary object HBR

* Hejduk, M.D. and Johnson, L.C. "Approaches to Evaluating Probability of Collision Uncertainty." 2015 AAS/AIAA Space Flight Mechanics Meeting, Napa CA, February 2015

Covariance Uncertainty: Evaluation Products

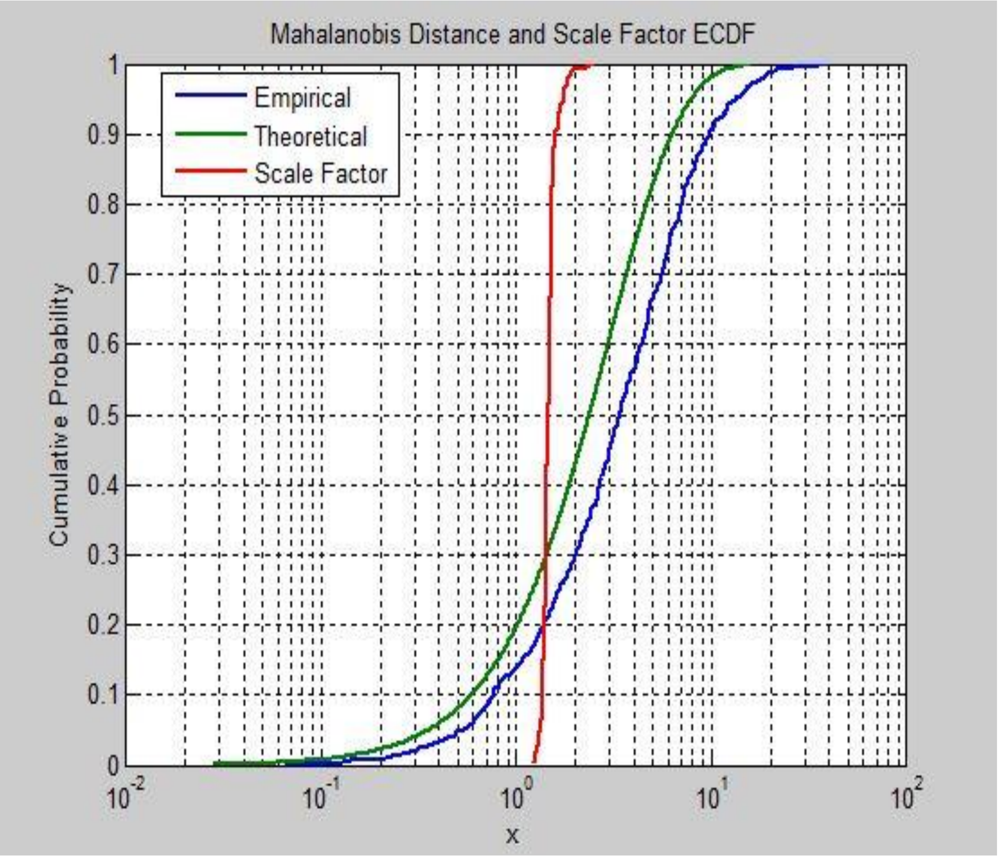
- **Covariance realism can be computed by comparing actual state errors and determining how well covariances represent these errors**
- JSpOC-resident utility generates reference orbits for every satellite
  - Covariance data from generating ODs preserved
- Second utility compares each generated SP vector to reference orbit at propagation points of interest
  - 1, 2, 3, 5, and 7 days from epoch
  - Calculates position residuals and combined covariance, which is combination of propagated vector covariance and reference orbit covariance
- With position residuals and combined covariance, can compute covariance “realism” factor for each vector at each prop point
  - For each vector, can calculate $\varepsilon^{T} C^{-1} \varepsilon$ (M^2, square of Mahalanobis distance)
    - ε is the vector of position residuals; C is the combined covariance
  - If covariance realistic, M^2 set should produce a 3-DoF chi-squared distribution

Covariance Uncertainty: Canonical and "Adjusted" Scale Factors

- In evaluating position states/covariances $\varepsilon^{T}C^{-1}\varepsilon$ has expected value of 3
- Thus, can represent covariance error by a scale factor by which the covariance would have to be multiplied in order to force the $\varepsilon^{T}C^{-1}\varepsilon$ value to 3
  - Would produce a set of scale factors whose PDF could be said to characterize the expected covariance errors
  - However, forces all cases to produce the mean value, which is not accurate for any distribution other than the uniform distribution
- Instead, produce "adjusted" scale factors:
  - Produce and rank-order set of scale factors described above
  - Align with corresponding percentile values from the 3-DoF χ^2 distribution
  - For each aligned pair, calculate factor needed to force the $\varepsilon^{T}C^{-1}\varepsilon$ value to the χ^2 value for that particular percentile point
  - Adjusted scale factors thus not the value to force each χ^2 calculation to the value of 3 but rather what is needed at each percentile level to force the calculation to equal the χ^2 distribution value for that percentile level

Example Scale Factor Computation

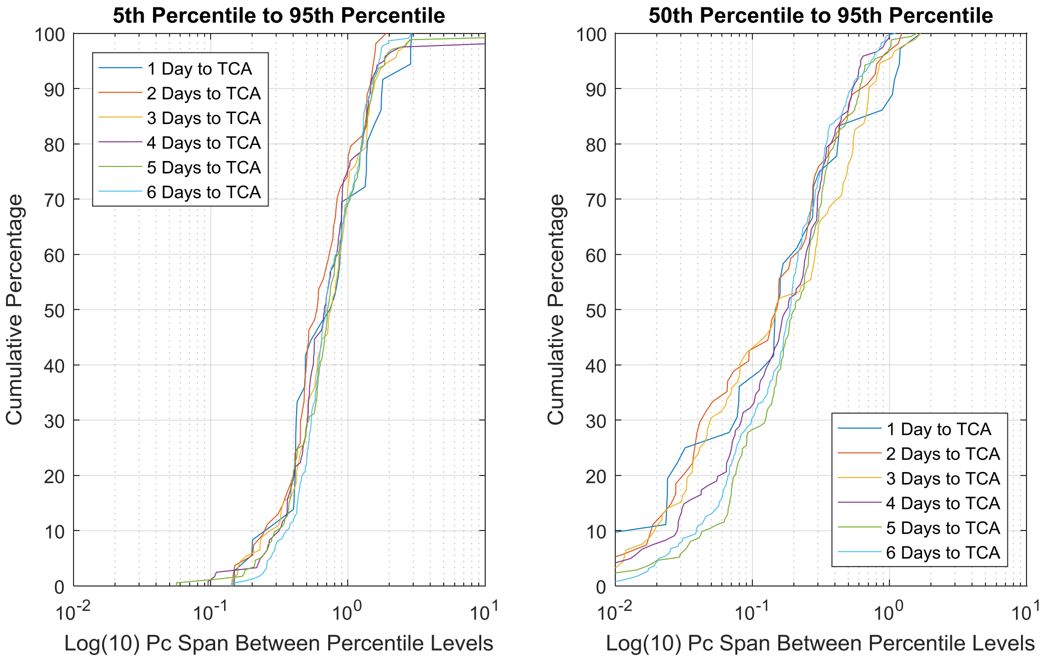
- Plot of a set of covariance evaluations for a single satellite shows
  - the empirical distribution of chi-squared variables
  - the theoretical distribution to which they should conform
  - the distribution of scale factors needed to make the empirical distribution match the theoretical one
- PDF of possible scale factors includes likelihood that each scale factor will arise
  - Pc values calculated from Monte Carlo draws from set of these scale factors will properly map the error likelihood into resultant Pc PDF
  - Avoids need for Pc Max construct



Test Dataset and Methodology

- Conjunction data for two satellites
  - NASA Earth-observation satellites Aqua and Aura
    - Near-circular, sun-synchronous orbits at ~705 km
  - Dataset span 1 MAY 2015 to 30 JUN 2016
    - Incorporates latest JSpOC service release
  - Slightly more than 9,000 conjunctions examined
- Prototype algorithm tested against this database
  - Determine what the PDFs of Pc values look like and how they compare to the nominal Pc values presently used in operations
  - Determine how the use of Pc PDFs would affect/enhance risk assessment decisions
- Divide conjunction events by color, to indicate severity
  - Green: not operationally worrisome; Pc < 1E-07
  - Red: operationally worrisome; Pc > ~1E-04
  - Yellow: has propensity to become worrisome; between green and red

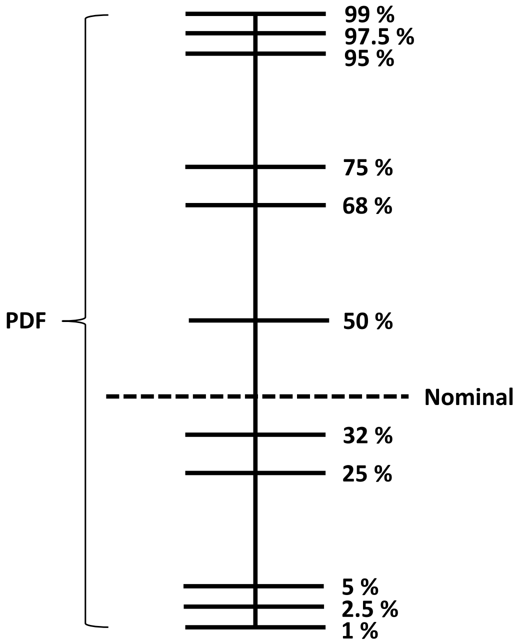
Results: PDF Span (Yellow)



- 5-to-95 span less than one OoM 70% of the time
- 50-to-95 span less than one OoM nearly all the time
- Red performance very similar—spans are tight
- Not all that much difference among different propagation states

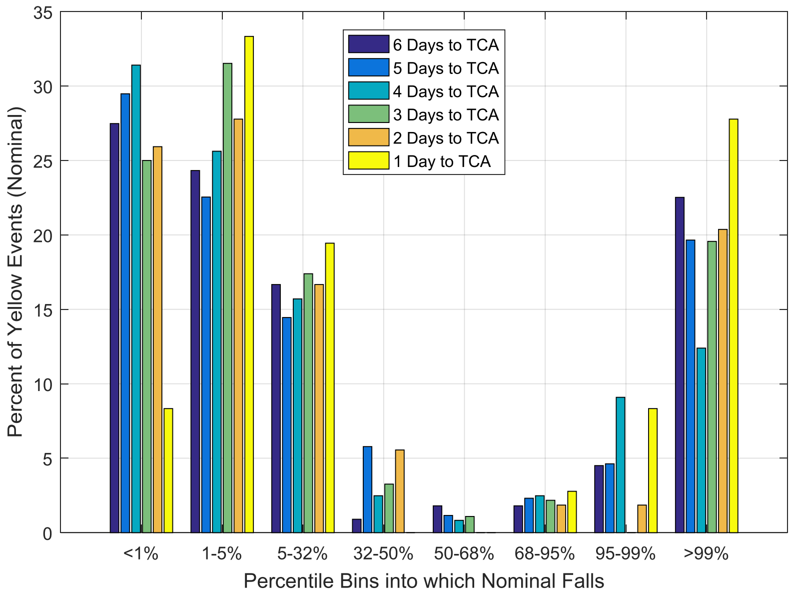
Results: Placement of Nominal

- What is alignment of nominal Pc with the Pc PDF?
- PDFs divided into percentile bins shown at right
- For each update, PDF bin into which nominal Pc falls is noted
  - In example at right, nominal falls into 32-50 percentile bin
  - Nominal can also stand entirely above or below PDF (end points defined as 1st and 99th percentile)
- Purpose is to determine how different nominal value is from associated PDF that considers covariance error



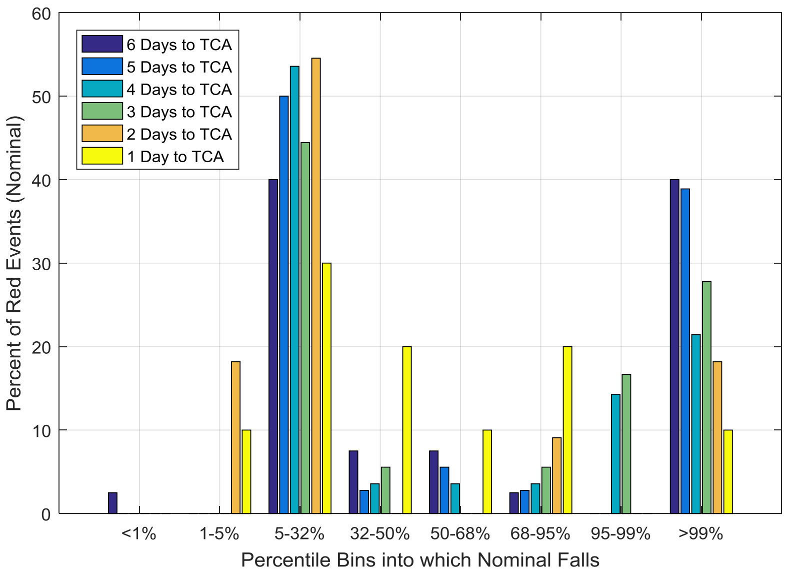
Placement of Nominal: Yellow Events

- Situation is bifurcated
  - > 50% of updates place nominal below PDF 5th percentile
    - PDF produces "higher" Pc
  - 25-30% of updates place nominal above PDF 95th percentile
    - PDF produces "lower" Pc
  - Less than one-third of updates between 5th and 95th percentiles
- Shows importance of considering covariance error
  - Can drive Pc strongly away from nominal in either direction

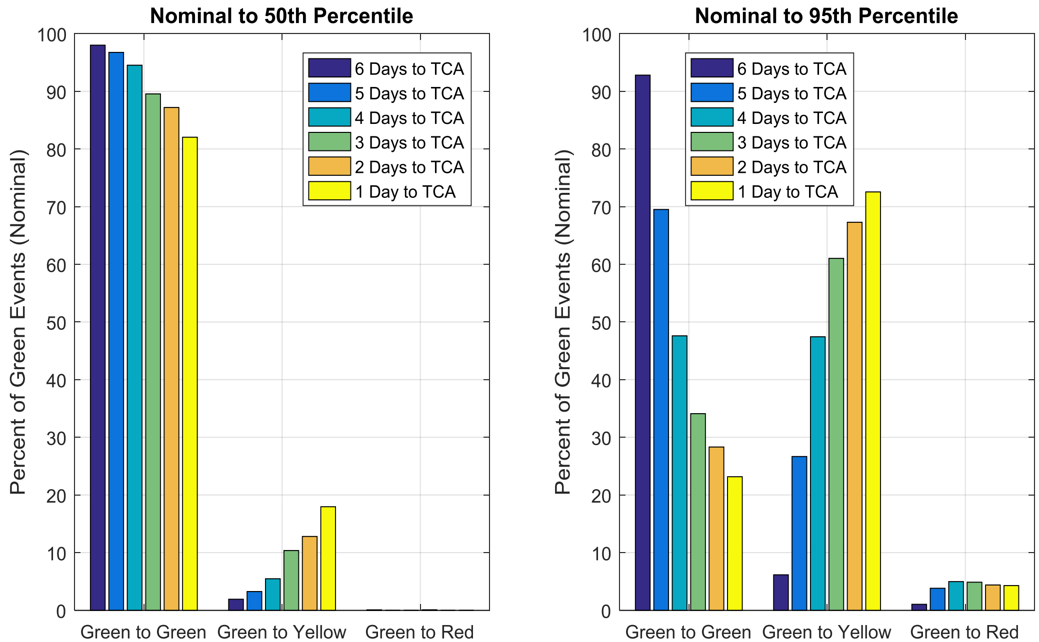


Placement of Nominal: Red Events

- Situation again bifurcated, but less so
  - At two days to TCA, 75% of Pc nominals < 32nd percentile
- Consideration of covariance error tends to increase Pc value for red events
  - Not as significant, since events already red
- 20-25% of situations put nominal entirely above PDF at 2-3 days to TCA
  - Reduction in event severity
- Like with yellow, error important because influence not in single direction

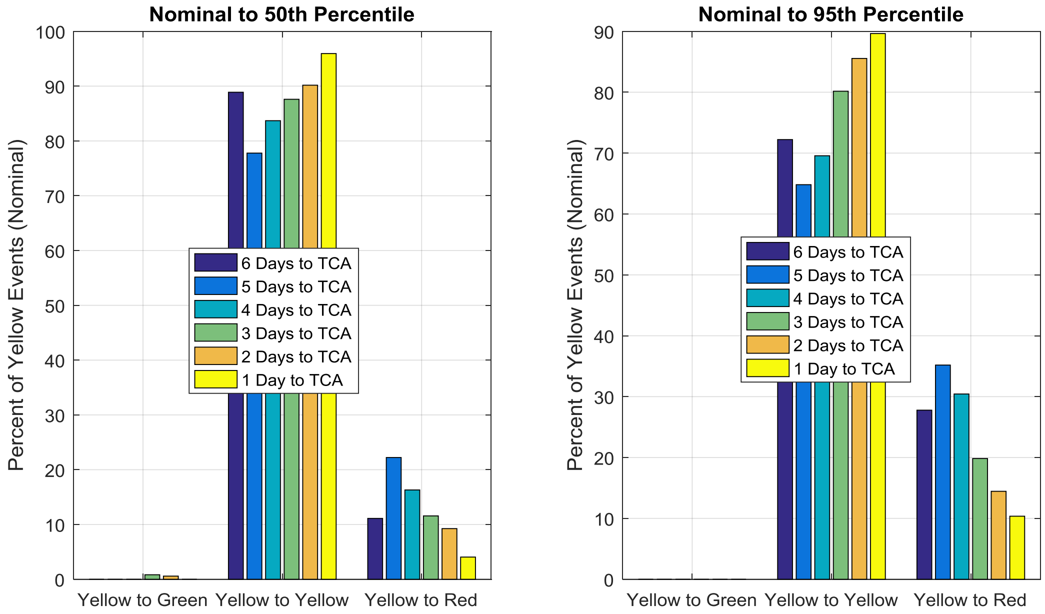


Change of Color (nominal vs PDF): Green Events



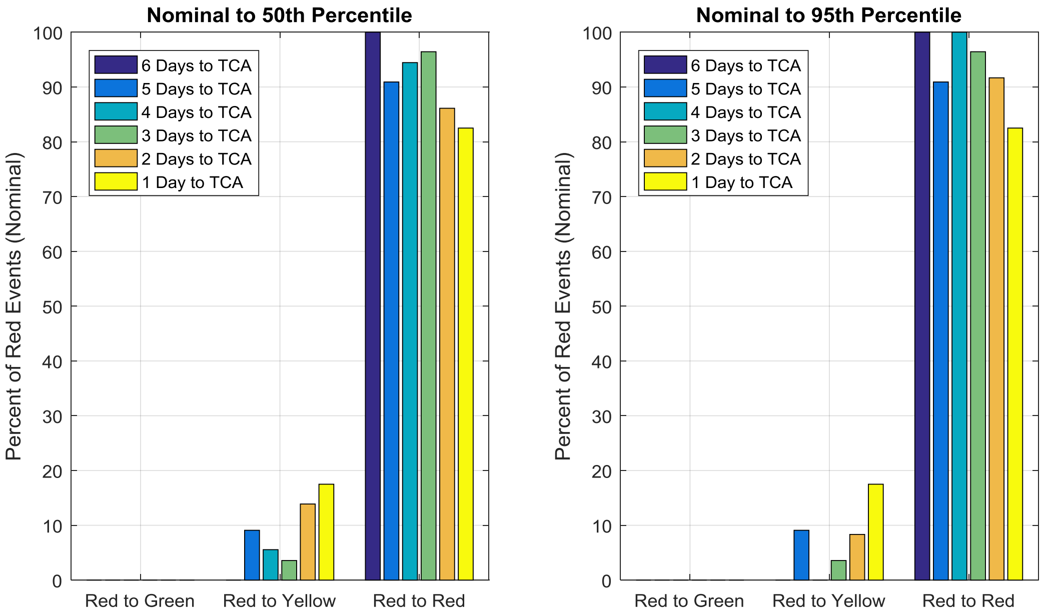
- Against 50th percentile, rather minor level of promotion (10%)
- Against 95th percentile, >50% of events promoted to yellow
- When covariance uncertainty considered, significantly more events considered worthy of expanded monitoring

Change of Color (nominal vs PDF): Yellow Events



- Against 50th percentile, rather minor level of promotion (10%)
- Against 95th percentile, ~20% of events promoted to red
- Virtually no events demoted to green
- Significant outcome: up to ~20% of yellow events become serious when covariance error included

Change of Color (nominal vs PDF): Red Events



- For either percentile point comparison, amount of red-to-yellow regression relatively small but not discountable (~10% of cases)

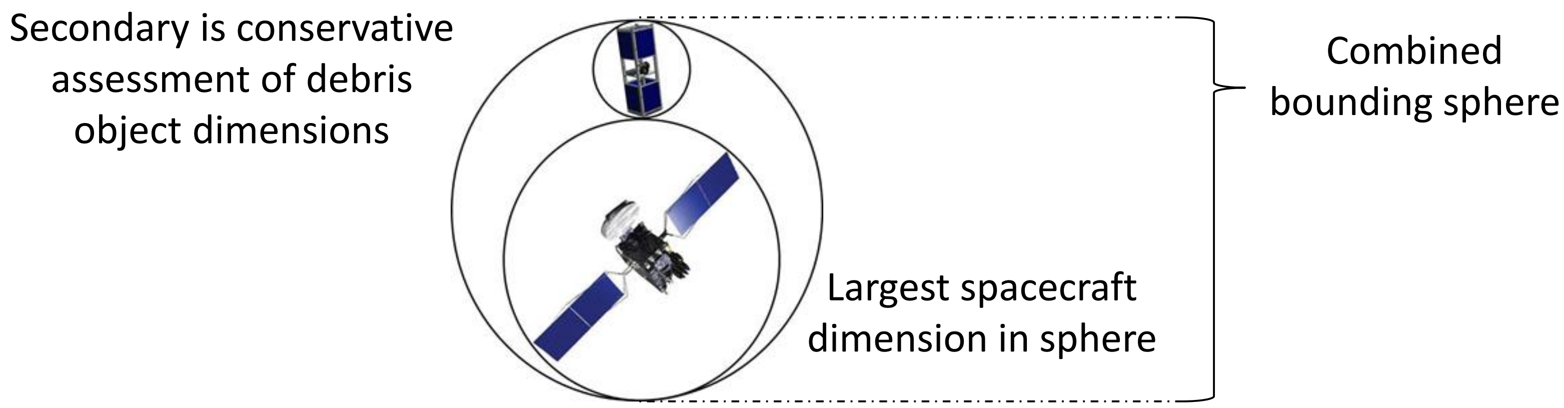
Covariance Uncertainty: Summary

- PDFs remain relatively tight and well-behaved
- Many cases in which PDF nearly to entirely above or below nominal
  - Both for yellow and red events
- Many of these cases result in an event color redesignation
  - Both promotion and demotion of color level

- Incorporation of covariance uncertainty can thus have a significant effect on CA operations
  - Will be incorporated into NASA operational paradigms

Hard-Body Radius

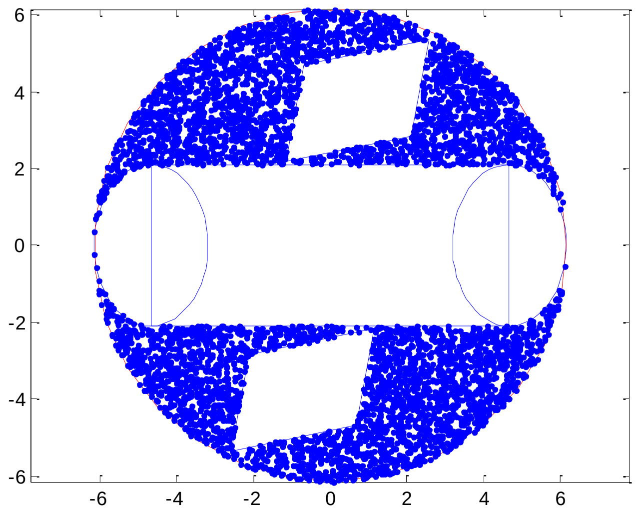
- HBR is typically determined by circumscribing primary and secondary objects in spheres and then combining into one bounding sphere
  - Size of the secondary is typically not known, so added as a large estimate of debris object dimensions



- HBR uncertainties that follow represent a more realistic estimate of the primary satellite's area in the conjunction plane
  - Much smaller than the bounding sphere

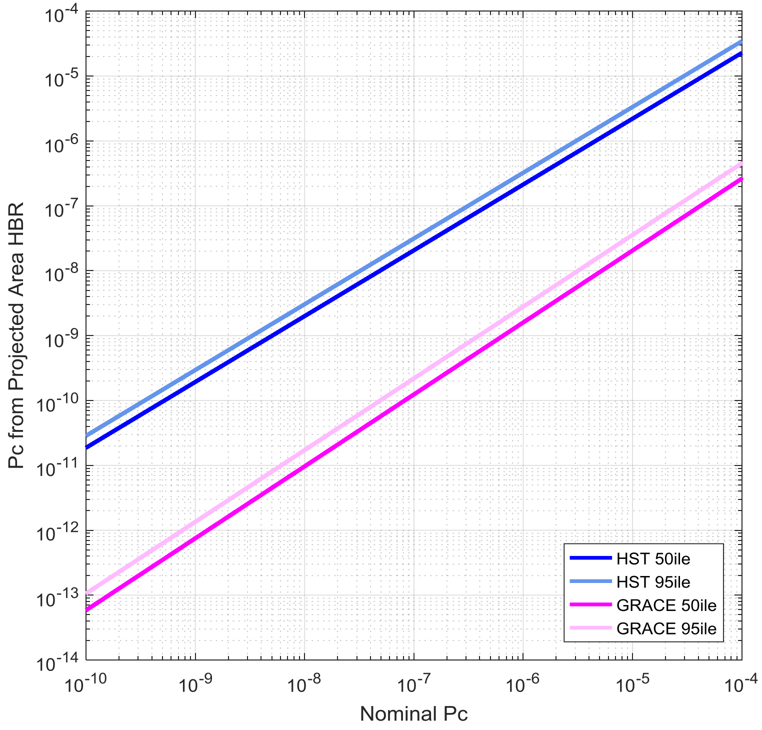
Methodology

- HBR alternative approach: projected area
  - Develop CAD satellite model
    - Can be simplified ball-and-stick variety
  - Project all possible rotations into plane
  - Create PDF of all these projected areas
- Examine differences in Pc values
  - Nominal using circumscribing spheres
  - New method using projected area PDFs
- Dataset for examination
  - Same period and pedigree as previous dataset
  - Different satellites: Hubble Space Telescope and GRACE
    - Relatively straightforward to assemble CAD models
    - About 4000 conjunctions analyzed



HBR Uncertainty Results: Pc Offsets

- Chart shows relationship between nominal Pc and Pc with HBR PDF
  - Blue Hubble ST; red GRACE satellite
  - Dark line 50ile; lighter line 95ile
- Offset values
  - HST: a little more than one OoM
  - GRACE: about 3 OoM
  - Compared to 10m and 20m HBRs, which are larger than necessary
- PDF percentiles have muted effect
  - Most of change from difference between large HBR and median
  - Reducing HBR to largest projected area value renders most of benefit



HBR Uncertainty Results: Changes in Event Severity

	Yellow to Green	Red to Yellow
50th Percentile Projected Area	41.1% of Yellow Events	100% of Red Events
95th Percentile Projected Area	35.1% of Yellow Events	91.2% of Red Events

- Because of bounding-sphere overstatement of HBR, all uses of projected area approach reduce Pc
  - Will not increase event color
- Yellow to green demotion either 35 or 41%, depending on percentile
- Red to yellow demotion either 91 or 100%, depending on percentile
- Numbers are significant; difference between percentile levels less so
- Reasonable operational implementation recommendation: use largest projected area value
  - Produces most of the offset and minimizes disagreements over details

Conclusions

- Both error sources, taken individually, can result in significant changes to the Pc and associated risk level of CA events
  - For covariance uncertainty, about 20% of yellow events can become red and 10% of red events can become yellow
  - For HBR uncertainty, > 50% of red events can be rendered yellow for the two satellite types examined
- Effect on operations considerable
  - Potential increase in serious events by 20% is significant
  - Ability to downgrade large numbers of events important *simpliciter* and to prepare for S-Band Fence, which could increase numbers of events substantially
- NASA CARA moving forward to implement these techniques in operations
  - Sample display on next slide

Pc Uncertainty: Sample Display